University of Wisconsin – Madison

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Finite width effects in Higgs decays as a means of measuring massive particle widths

D.J. Summers*

Department of Physics,
University of Wisconsin – Madison,
1150 University Avenue,
Madison,
WI 53706,
U.S.A.

Abstract

We calculate decays of a Standard Model Higgs boson to a virtual massive particle and discuss how this depends on the massive particle total width. If the partial width of Higgs to a virtual massive particle can be measured this gives a measurement of that massive particle's width. We discuss how one would go about measuring these partial widths of a Higgs experimentally, and how this could lead to a measurement of the W boson and t quark width. For the latter extreme dependence on the Higgs mass and the small $H \to tt^*$ branching ratios mean that little can be learnt about the t quark width. For the former there is also large dependence on the Higgs mass; however this can be removed by taking the ratio of $H \to WW^*$ decays to $H \to ZZ^*$ decays. This ratio also has the advantage of being fairly insensitive to physics beyond the Standard Model. Unfortunately, for Higgs masses of interest the $H \to ZZ^*$ branching ratio is small enough that we require many 1000's of tagged Higgs decays before an accurate measurement of the W width can be made. This is likely to be hard experimentally.

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* Email address: summers @ phenxr.physics.wisc.edu

1 Introduction

The W and Z bosons that we observe in high energy particle colliders are massive objects. This mass does not enter our theoretical models at a fundamental level, but instead is typically generated by some spin zero object. The physical existence of this spin zero object is among the most important questions that we have about physical reality – and if this particle is discovered then its properties will become of prime interest. In the Standard Model this spin zero object is also a fundamental field of the theory, the Higgs boson. In this paper we examine how the properties of this Higgs boson may be used to probe other aspects of Standard Model physics; in particular we discuss how Higgs decays to off-shell massive particles may be used as a probe of that particles width.

If a particle has a mass below the threshold to decay to on-shell massive particles, then that decay can still occur through the decay to off-mass-shell particles; although the decay rate is usually very small due to the off-mass shell propagator. However in the Standard Model the Higgs boson is responsible for generating all particle masses. So the more massive the particle the stronger the Higgs couples to that particle. This means that Higgs decays to off-shell massive particles, although suppressed by the off-shell propagator, are enhanced by the strong Higgs coupling. Thus Higgs decays to off-shell particles can be appreciable, this can be seen especially in the case $H \to W^{(*)}W^{(*)}$ where the branching ratio is above 10% for Higgs masses as low as 115 GeV despite this forcing a W at least 45 GeV off mass shell.

It is interesting to consider how a two stage decay takes place; that is a decay that proceeds via an intermediate particle,

$$A \to B \to C \qquad . \tag{1.1}$$

At leading order (LO) the decay rate for this looks like

$$\Gamma(A \to B \to C) \sim \int \frac{d(p^2)}{(p^2 - m_B^2)^2 + m_B^2\Gamma_B^2} \int d(\mathrm{LIPS}_{B\to C})|\mathcal{M}|^2 \qquad . \tag{1.2}$$

Now when the B width, Γ_B, is small the integral over p^2 can be done. When we are above threshold the narrow width approximation gives

$$\Gamma(A) \sim \frac{\pi}{m_B\Gamma_B} \int d(\mathrm{LIPS}_{B\to C})|\mathcal{M}|^2 \qquad . \tag{1.3}$$

This appears to diverge in the limit $\Gamma_B \to 0$; however the integral over the B to C phase space gives a term

$$\int d(\mathrm{LIPS}_{B\to C})|\mathcal{M}|^2 \sim \Gamma_B^{\mathrm{LO}}\mathrm{Br}(B \to C) \qquad , \tag{1.4}$$

and so in the narrow width approximation we find

$$\Gamma(A \to B \to C) \simeq \Gamma(A \to B)\frac{\Gamma_B^{\rm LO}}{\Gamma_B}{\rm Br}(B \to C) \qquad . \tag{1.5}$$

This is the result we expect as long as $\Gamma_B^{\rm LO} = \Gamma_B$, that is that the B width in the Breit-Wigner propagator is the same as the width that comes from integrating the matrix element over the B decay phase space. We could of course choose Γ_B equal to $\Gamma_B^{\rm LO}$; however as we include higher order corrections to the decay of B we expect that the $\Gamma_B^{\rm LO}$ in the numerator will tend towards the physical value of the width, Γ_B. As such in this work we will always replace the width in the numerator by the physical width, Γ_B.

If we now consider the case where the decay of A via an on-shell B is kinematically forbidden, then A can still decay via a virtual B^*,

$$A \to B^* \to C \qquad . \tag{1.6}$$

Then in the narrow width approximation the integral over the Breit Wigner becomes

$$\int \frac{d(p^2)}{(p^2-m_B^2)^2 + m_B^2\Gamma_B^2} \to \int \frac{d(p^2)}{(p^2-m_B^2)^2} \qquad , \tag{1.7}$$

so this integral is no longer proportional to $1/\Gamma_B$; however we still get a Γ_B in the numerator of the width from the integral of the matrix element over the B phase space. This means that

$$\Gamma(A \to B^*) \sim \Gamma(B) \qquad , \tag{1.8}$$

and so measuring the decay width of A into a virtual B gives us information about the B width, or more strictly the running B^* width, which is related to the B width.

In this paper we calculate the decays of the Standard Model Higgs boson that proceed via a massive virtual particle. In the Standard Model there are 3 very massive particles, the W and Z bosons and the t quark; we consider all Higgs decays that involve these massive particles,

$$H \to Z^{(*)}\gamma \to f_1\bar{f}_1\gamma \tag{1.9}$$

$$H \to Z^{(*)}Z^{(*)} \to f_1\bar{f}_1\ f_2\bar{f}_2 \tag{1.10}$$

$$H \to W^{(*)}W^{(*)} \to f_1\bar{f}_2\ f_3\bar{f}_4 \tag{1.11}$$

$$H \to t^{(*)}\bar{t}^{(*)} \to bW^{(*)}\bar{b}W^{(*)} \to bf_1\bar{f}_2\ \bar{b}f_3\bar{f}_4 \tag{1.12}$$

The rates for processes (1.10,1.11,1.12) have been calculated before in Ref. 1, the rate for process (1.9) has not previously been published[2].

2 Breit–Wigner propagators

As we are interested in measuring massive particle widths in this paper, and a large dependence on the particles width arises from the Breit-Wigner propagator, we need to consider the form of the Breit-Wigner propagator that we use. If we start with a bare massive spin 0 particle propagator and we sum an arbitrary number of one particle irreducible insertions we have,

$$\begin{aligned}\text{Prop} &= \frac{1}{p^2-m_0^2}+\frac{1}{p^2-m_0^2}\Pi(p^2)\frac{1}{p^2-m_0^2}+\frac{1}{p^2-m_0^2}\Pi(p^2)\frac{1}{p^2-m_0^2}\Pi(p^2)\frac{1}{p^2-m_0^2}+\ldots \\ &= \frac{1}{p^2-m_0^2-\Pi(p^2)} \\ &= \frac{1}{p^2-(m_0^2+\text{Re}(\Pi(p^2)))-i\text{Im}(\Pi(p^2))} \\ &= \frac{1}{p^2-m_R^2(p^2)-i\text{Im}(\Pi(p^2))} \qquad , \end{aligned} \tag{2.1}$$

where the real part of the one particle irreducible diagrams have been reabsorbed into the definition of the particle mass to give a running mass. The running mass is then related to the physical pole mass through the relationship

$$m_R^2(m^2) = m^2 \qquad . \tag{2.2}$$

In this work we will not calculate the real part of the one particle irreducible diagrams at all, and will always use a fixed mass,

$$m_R^2(p^2) = m^2 \qquad . \tag{2.3}$$

Now the imaginary part of the one particle irreducible diagrams is related to the total width through the optical theorem

$$\begin{aligned}\text{Im}(\Pi(p^2)) &= -\frac{1}{2}\int d(\text{LIPS})|\mathcal{M}_{\text{decay}}|^2 \\ &= -m\Gamma(p^2) \qquad , \end{aligned} \tag{2.4}$$

where we have defined the running width as

$$\Gamma(q^2) = \frac{1}{2m}\int d(\text{LIPS}_{\text{q}^2})|\mathcal{M}_{\text{decay}}|^2 \qquad . \tag{2.5}$$

Notice that we use $1/2m$ as the flux factor for all q^2 values, whereas $\int d(\text{LIPS}_{\text{q}^2})|\mathcal{M}_{\text{decay}}|^2$ is evaluated for a particle with $p^2 = q^2$ to decay.

This gives the Breit–Wigner propagator

$$\mathrm{Prop}_0 = \frac{1}{p^2 - m^2 + im\Gamma(p^2)} \quad . \tag{2.6}$$

If we consider the propagator of a massive spin 1 gauge boson, then if that particle only decays to massless fermions (as we will consider in this paper) then we only have a contribution from the transverse part of the propagator. This gives the form of the propagator as

$$\mathrm{Prop}_1 = \frac{-g^{\mu\nu} + p^\nu p^\nu/p^2}{p^2 - m^2 + im\Gamma(p^2)} \quad , \tag{2.7}$$

where the $p^\nu p^\nu/p^2$ term in the numerator always cancels on massless fermions. If the spin 1 particle only decays into massless particles then, neglecting the running of coupling constants, we know that $m\Gamma(p^2) \sim p^2$ and so we can write the propagator in terms of the on-mass-shell width,

$$\mathrm{Prop}_1 = \frac{-g^{\mu\nu} + p^\nu p^\nu/p^2}{p^2 - m^2 + ip^2\Gamma(m^2)/m} \quad . \tag{2.8}$$

The Breit Wigner for a massive fermion is worth considering in more detail. The imaginary part of the one particle irreducible diagrams come from the diagrams where the heavy quark decays into a light quark through the emission of a W boson. The one particle irreducible insertion, without the heavy quark propagators, is given by

$$\Pi = B(p^2)\not{p}(1-\gamma_5) \quad , \tag{2.9}$$

where p is the heavy quark momentum and B is a scalar function of p^2 only. Multiplying by $(\not{p}+m)$ and taking the trace gives

$$4p^2\mathrm{Im}(B) = \mathrm{Im}(B)\mathrm{Tr}((\not{p}+m)\not{p}(1-\gamma_5)) = \mathrm{Im}(\mathrm{Tr}((\not{p}+m)\Pi)) = -2m\Gamma \quad , \tag{2.10}$$

where the last equality is given by the optical theorem. This gives

$$\mathrm{Im}(B) = -\frac{m\Gamma}{2p^2} \quad . \tag{2.11}$$

After resumming the one particle irreducible diagrams the heavy quark propagator is given by

$$\begin{aligned}\mathrm{Prop}_{1/2} &= \frac{1}{\not{p}-m} + \frac{1}{\not{p}-m}\Pi\frac{1}{\not{p}-m} + \frac{1}{\not{p}-m}\Pi\frac{1}{\not{p}-m}\Pi\frac{1}{\not{p}-m} + \ldots \\ &= \frac{1}{\not{p} - m - B\not{p}(1-\gamma_5)} \\ &= \frac{\not{p} + m - B\not{p}(1-\gamma_5)}{p^2(1-2B) - m^2} \quad .\end{aligned} \tag{2.12}$$

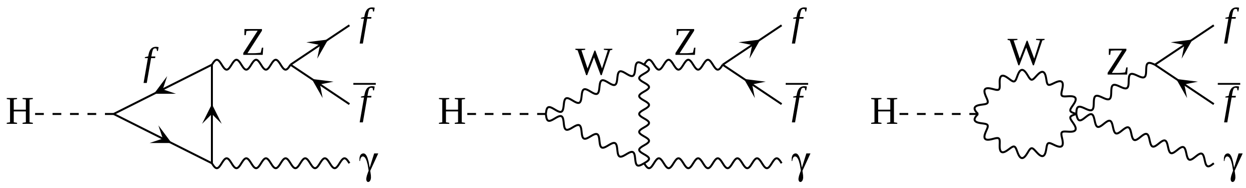

Fig. 1 The Feynman diagrams for the process $H \to Z^*\gamma \to f\bar{f}\gamma$.

Now if either end of the heavy quark propagator couples onto a W boson then the $\not{p}(1-\gamma_5)$ in the numerator cancels directly. As in this paper we decay all top quarks, and so one end of the propagator is always coupled to a W, we drop the $\not{p}(1-\gamma_5)$ term in the numerator. Then the the imaginary part of B gives the fermion Breit–Wigner as

$$\mathrm{Prop}_{1/2} = \frac{\not{p} + m}{p^2 - m^2 + im\Gamma(p^2)} \quad . \tag{2.13}$$

Although this is the naive form of the Breit–Wigner fermion propagator we notice that it disagrees with the form of the Breit–Wigner fermion propagator given in Ref. 3. Also notice that this form of the propagator is different from Ref. 1 where they use $1/2\sqrt{q^2}$ as the flux factor in the definition of an off-shell decay width, whereas we use $1/2m$.

3 Higgs decay widths

In this section we give the decay rates for a Higgs to decay in the channels (1.9–1.12). Now the decay (1.9) does not occur at tree level, and so the lowest order diagrams are at the 1 loop level and shown in Fig. 1. At the same order in perturbation theory there are other Feynman diagrams that also contribute to the decay $H \to f\bar{f}\gamma$, that do not proceed via a single Z boson. In this work we will calculate just the diagrams associated with the decay $H \to Z^*\gamma$, even though this does not give the full rate for $H \to f\bar{f}\gamma$, or is even gauge invariant with respect to the $SU(2)$ gauge. This means that if we find any interesting physics associated with the decay $H \to Z^*\gamma$ then a more complete calculation needs to be done. As the result for (1.9) has not been previously published we give a fairly complete derivation of this result. We also give the partial widths for (1.10–1.12), although these have been calculated before [1], we give the results again here. Our results differ slightly from Ref. 1, as we have defined our running widths in a slightly different way, and hence use a different form of the running width in the Breit–Wigner propagator. Our form of the Breit–Wigner propagator is motivated by the use of the optical theorem and so we expect it to be more accurate than that used in Ref. 1.

The Feynman diagrams for $H \to Z^{(*)}\gamma \to f_1\bar{f}_1\gamma$ are shown in Fig. 1, it is convenient with this process to split the calculation up into two halves, into the process $H \to Z^*\gamma$ followed by the process $Z^* \to f\bar{f}$. The calculation of the process $H \to Z^*\gamma$ is identical to the calculation of $H \to Z\gamma$, which has been done many times before. See for example Ref. 4. Here we do not repeat the calculation but just quote the results. The effective coupling for $HZ\gamma$ vertex is given by

$$\mathcal{M}^{\mu\nu}_{HZ\gamma} = A(p^{\mu}_{Z}p^{\mu}_{\gamma} - p_Z\cdot p_\gamma\, g^{\mu\nu}) \qquad , \tag{3.1}$$

where A has the form [4]

$$A = \frac{\alpha g}{4\pi M_W}(A_F + A_W) \tag{3.2}$$

and A_F and A_W are given by

$$A_F = \sum_{\text{fermions}} n_c \frac{-2e_f(T^3_f - 2e_f \sin^2\theta_W)}{\sin\theta_W \cos\theta_W}[I_1(\tau_f, \lambda_f) - I_2(\tau_f, \lambda_f)] \tag{3.3}$$

and

$$\begin{aligned} A_W = &- \cot\theta_W \Bigg\{ 4(3 - \tan^2\theta_W) I_2(\tau_W, \lambda_W) \\ &+ \left[\left(1 + \frac{2}{\tau_W}\right)\tan^2\theta_W - \left(5 + \frac{2}{\tau_W}\right)\right] I_1(\tau_W, \lambda_W)\Bigg\} \qquad , \end{aligned} \tag{3.4}$$

where,

$$\tau_f \equiv \frac{4m_f^2}{m_H^2} \qquad \lambda_f \equiv \frac{4m_f^2}{p_Z^2} \qquad \tau_W \equiv \frac{4m_W^2}{m_H^2} \qquad \lambda_W \equiv \frac{4m_W^2}{p_Z^2} \qquad , \tag{3.5}$$

with,

$$\begin{aligned} I_1(a,b) &= \frac{ab}{2(a-b)} + \frac{a^2b^2}{2(a-b)^2}[f(a) - f(b)] + \frac{a^2b}{(a-b)^2}[g(a) - g(b)] \\ I_2(a,b) &= -\frac{ab}{2(a-b)}[f(a) - f(b)] \end{aligned}$$

$$f(\tau) = \begin{cases} \left[\sin^{-1}\left(\sqrt{1/\tau}\right)\right]^2 & \text{if } \tau \geq 1, \\ -\frac{1}{4}[\ln(\eta_+/\eta_-) - i\pi]^2 & \text{if } \tau < 1, \end{cases} \tag{3.6}$$

$$g(\tau) = \begin{cases} \sqrt{\tau - 1}\sin^{-1}(1/\sqrt{\tau}) & \text{if } \tau \geq 1, \\ \frac{1}{2}\sqrt{\tau - 1}[\ln(\eta_+/\eta_-) - i\pi] & \text{if } \tau < 1, \end{cases} \tag{3.7}$$

$$\eta_{\pm} \equiv (1 \pm \sqrt{1 - \tau}) \qquad . \tag{3.8}$$

This leads to the matrix element squared

$$|\mathcal{M}|^2 = 8(g_V^2 + g_A^2)\frac{|A|^2 g_Z^2}{\mathrm{BW}_Z(p_Z^2)}\, p_f\cdot p_{\bar{f}}\, \left((p_\gamma\cdot p_f)^2 + (p_\gamma\cdot p_{\bar{f}})^2\right) \qquad , \tag{3.9}$$

where

$$\mathrm{BW}_V(p_V^2) \equiv (p_V^2 - M_V^2)^2 + M_V^2\Gamma_V^2(p_V^2) \qquad . \tag{3.10}$$

The partial width is given by

$$d\Gamma = (2\pi)^4 \frac{1}{2M_H}\delta^4\left(p_H - \sum p_{\mathrm{final}}\right)|\mathcal{M}|^2 \prod \frac{d^3p_{\mathrm{final}}}{2E_{\mathrm{final}}(2\pi)^3} \qquad . \tag{3.11}$$

Most of these integrations can be done analytically, leaving the integration over the Z line shape

$$\Gamma(H \to Z^*\gamma) = \frac{\Gamma_Z M_H^3}{32\pi^2 M_Z}\int_0^{M_H^2} dp_Z^2 \left(1 - \frac{p_Z^2}{M_H^2}\right)^3 \frac{A^2 p_Z^2}{\mathrm{BW}_Z(p_Z^2)} \qquad , \tag{3.12}$$

where we have replaced the $M_Z/(12\pi)g_Z^2(g_V^2 + g_A^2)$ in the numerator with the on-shell non-running Z width. This last integral over the Z line shape is best done numerically.

The partial width for $H \to V^{(*)}V^{(*)}$ is given by

$$\Gamma(H \to V^*V^*) = \frac{\Gamma_V^2}{M_V^2}\frac{g_{HVV}^2}{64\pi^3 M_H}\int \frac{d(p_{V_1}^2)}{\mathrm{BW}_{V_1}(p_{V_1}^2)}\frac{d(p_{V_2}^2)}{\mathrm{BW}_{V_2}(p_{V_2}^2)}((M_H^2 - p_{V_1}^2 - p_{V_2}^2)^2 + 8p_{V_1}^2 p_{V_2}^2)$$
$$\times\sqrt{1 - 2(p_{V_1}^2 + p_{V_2}^2)/M_H^2 + (p_{V_1}^2 - p_{V_2}^2)^2/M_H^4} \qquad . \tag{3.13}$$

In this expression we have again again collected the terms in the numerator that give the on-shell non-running vector boson width, so again we are not restricted to use the LO value, but can use the physical value.

Now for the $H \to Z_1^{(*)}Z_2^{(*)} \to f_1\bar{f}_1 f_2\bar{f}_2$ we need to include a symmetry factor of $\frac{1}{2}$. This is because when $f_1 \neq f_2$ we over count the decays of the Z's by counting both $Z_1 \to f_1\bar{f}_1$, $Z_2 \to f_2\bar{f}_2$ and $Z_1 \to f_2\bar{f}_2$, $Z_2 \to f_1\bar{f}_1$ despite the fact that these are both the same decay as there is no distinction between Z_1 and Z_2. When $f_1 = f_2$ then the symmetry factor comes from having identical particles in the final state.

For off-shell decays of W and Z bosons we will have interference between the decays $H \to W^{(*)}W^{(*)}$ and $H \to Z^{(*)}Z^{(*)}$, however these interference effects will only be large when both vecor bosons are forced off mass shell. Now when both vector bososns are forced off mass shell the Higgs partial widths will be exceptionally small and so the interference only contributes a small term. Hence we ignore this interference in this paper.

The $H \to t^{(*)}\bar{t}^{(*)}$ partial width is given by

$$\Gamma(H \to t^{(*)}\bar{t}^{(*)}) = N_c \frac{g_{ttH}^2 m_t^4}{8\pi^3 M_H} \int \frac{d(p_t^2)}{\mathrm{BW}_t(p_t^2)} \frac{d(p_{\bar{t}}^2)}{\mathrm{BW}_{\bar{t}}(p_{\bar{t}}^2)} \frac{\Gamma_t(p_t^2)}{p_t^2} \frac{\Gamma_t(p_{\bar{t}}^2)}{p_{\bar{t}}^2} \times \left((M_H^2 - p_t^2 - p_{\bar{t}}^2)(p_t^2 + p_{\bar{t}}^2)/2 - 2p_t^2 p_{\bar{t}}^2\right) \times \sqrt{1 + (p_t^2 - p_{\bar{t}}^2)^2/M_H^4 - 2(p_t^2 + p_{\bar{t}}^2)/M_H^2}, \tag{3.14}$$

where the tree level running t width is given by

$$\Gamma_t^{\mathrm{LO}}(p_t^2) = \frac{g_W^2 \Gamma_W}{8\pi^2 M_W m_t} \int_0^{p_t^2} \frac{d(p_W^2)}{\mathrm{BW}_W(p_W^2)} (p_t^4 - 2p_W^4 + p_W^2 p_t^2)(1 - p_W^2/p_t^2). \tag{3.15}$$

Notice that this integral can be done analytically; however the form is not particularly illuminating and so we do not give the result here, although we do use the analytic result in all results. As the t quark decays via a massive W boson this introduces a second scale into the problem. This means that unlike the case for W and Z where we know that for massless decays $m\Gamma \sim p^2$ there is no such simple relationship for Γ_t. In this paper we use

$$\Gamma_t(p^2) = \frac{\Gamma_t(m_t^2)}{\Gamma_t^{\mathrm{LO}}(m_t^2)} \Gamma_t^{\mathrm{LO}}(p^2) \quad . \tag{3.16}$$

4 Numerical results

In this section we give the numerical results that we obtain for the formulas in the previous section. In order to present branching ratios of the Higgs boson we require all partial widths of the Higgs; as we have only calculated a subset in the previous section we use the remainder from Ref. 5. To begin with we show the Higgs branching ratio as a function of the Higgs mass in Fig. 2. For this we use,

$$\begin{aligned} m_t &= 175\,\mathrm{GeV} & \Gamma_t &= \Gamma_t^{\mathrm{LO}} = 1.53\,\mathrm{GeV} \\ M_Z &= 91.187\,\mathrm{GeV} & \Gamma_Z &= 2.490\,\mathrm{GeV} \\ M_W &= 80.22\,\mathrm{GeV} & \Gamma_W &= 2.08\,\mathrm{GeV} \quad . \end{aligned} \tag{4.1}$$

It is clear that for Higgs masses below $2M_W$ and $2M_Z$ that the branching ratios for $H \to WW^{(*)}$ and $H \to ZZ^{(*)}$ are still significant. The $H \to Z^{(*)}\gamma$ is only significantly different to the decay to a stable Z for Higgs masses below 100 GeV where the branching ratio is always less than 10^{-4} and so is not likely to be of experimental interest. As the t quark has turned out to be relatively heavy the Higgs branching ratio is always dominated

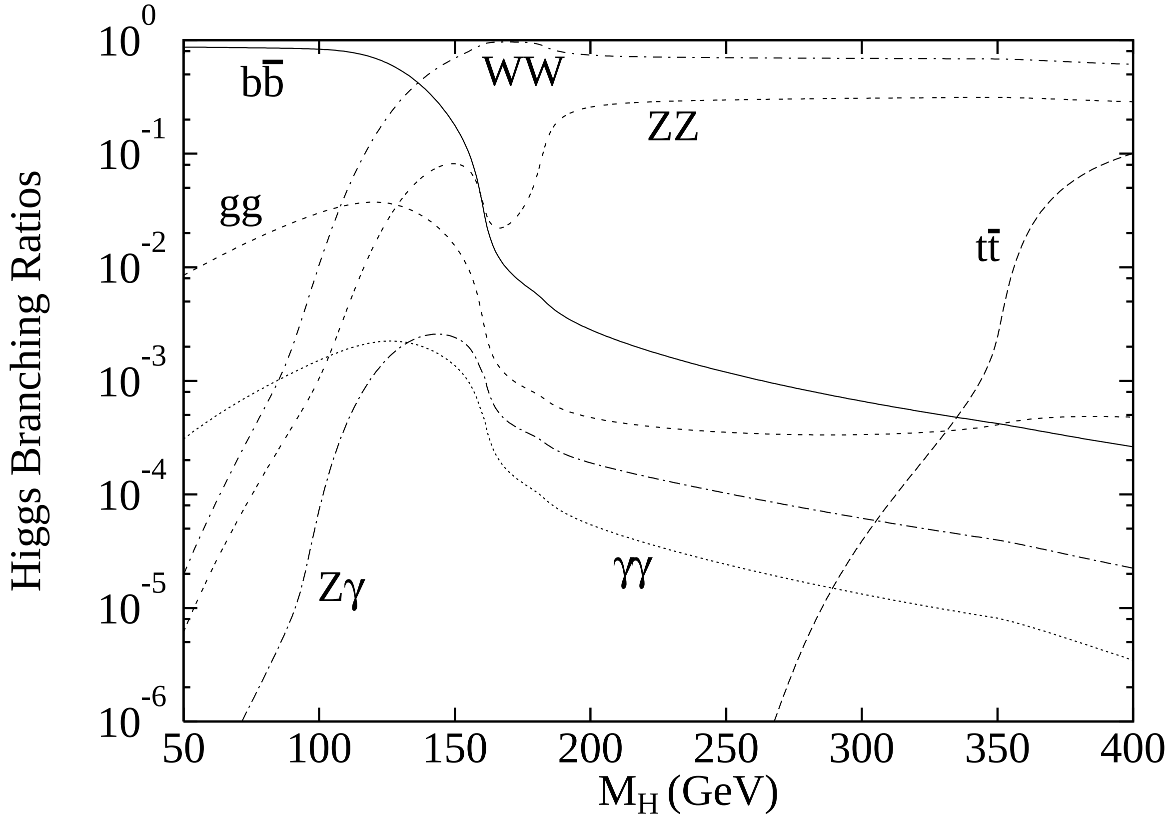

Fig. 2 Higgs branching ratios.

by the WW and ZZ decays. This, and that the t width is relatively narrow, means that when a t quark is forced off shell the t^*t branching ratio is always smaller than 10^{-3} and so it is likely to be exceptionally hard to experimentally measure.

If we now look at the dependence of these branching ratios on the width of massive particles, clearly the Z width is already known very accurately from LEP, and so Higgs decays will not improve the accuracy of this. The W width is currently measured at hadron colliders using two different methods. In the first the ratio of dilepton Z events is compared to single lepton + missing transverse energy W events. We have

$$\frac{\sigma(pp \to W \to l\nu)}{\sigma(pp \to Z \to ll)} = \frac{\sigma(pp \to W)}{\sigma(pp \to Z)} \frac{\mathrm{Br}(W \to l\nu)}{\mathrm{Br}(Z \to ll)} = \frac{\sigma(pp \to W)}{\sigma(pp \to Z)} \frac{\Gamma_Z}{\Gamma_W} \frac{\Gamma(W \to l\nu)}{\Gamma(Z \to ll)}. \qquad (4.2)$$

Now $\frac{\sigma(pp\to W)}{\sigma(pp\to Z)}$ and $\frac{\Gamma(W\to l\nu)}{\Gamma(Z\to ll)}$ can be well predicted within perturbation theory; Γ_Z is accurately measured at LEP, and so this gives a measurement of Γ_W. Of course this assumes that $\frac{\sigma(pp\to W)}{\sigma(pp\to Z)}$ and $\frac{\Gamma(W\to l\nu)}{\Gamma(Z\to ll)}$ can be accurately predicted, which is in turn based upon assumptions like the Standard Model being correct.

A more direct method is to look at the shape of the transverse mass distribution of W events. If the transverse mass is above the W mass then the decaying W is forced

above mass shell; whereas if the transverse mass is less than the W mass the dominant cross-section comes from on-shell W decays. Hence, just as with Higgs decays to virtual W's, the tail of the transverse mass distribution is sensitive to the W width. Thus a measurement of the shape of this tail gives a direct measurement of the W width. However the experimental errors that arise from this more direct method are far larger than the indirect first method. CDF finds [6, 7]

$$\Gamma_W^{\text{indirect}} = 2.063 \pm 0.061(\text{stat.}) \pm 0.060(\text{sys.}) \tag{4.3}$$

$$\Gamma_W^{\text{direct}} = 2.11 \pm 0.28(\text{stat.}) \pm 0.16(\text{sys.}) \qquad , \tag{4.4}$$

and so the W width is not particularly accuracy measured, especially in direct channels.

The t quark width is at the moment totally unmeasured, and there currently seems little prospect of measuring it at future colliders.

As such we will concentrate on what we can learn about W and t widths. As we have already mentioned in order to be sensitive to a massive particle width we need to be beneath the threshold to produce that particle, as such we shall choose 2 particular Higgs masses to study the W width and t width. For the W case we shall consider $M_H = 150\,\text{GeV}$. The WW branching ratio of the Higgs is significant for Higgs masses above $\gtrsim 110\,\text{GeV}$, so the Higgs branching ratio will have a similar sensitivity for all Higgs values between $110\,\text{GeV} \lesssim M_H < 2M_W$. For the t quark case we will consider $M_H = 350\,\text{GeV}$ ($= 2m_t$), this is the largest value of M_H that shows a significant dependence on Γ_t, yet the branching ratio is only 0.24%; for smaller values of M_H where the Higgs branching ratio still displays dependence on Γ_t the $H \to tt^{(*)}$ branching ratio drops rapidly. Also as $M_H = 2m_t$ is within the threshold region for the tt decay $\Gamma(H \to t^{(*)}t^{(*)}) \not\sim \Gamma_t$, and we have a less simple relationship.

Considering first the case where $M_H = 150\,\text{GeV}$, in Fig. 3 we show the partial width $\Gamma(H \to W^{(*)}W^{(*)})$ as a function of the on-shell W width. The dependence $\Gamma(H \to W^*W) \sim \Gamma(W)$ is clear to see, and so a measurement of $\Gamma(H \to W^*W)$ to say 10% accuracy gives a measurement of Γ_W to the same accuracy.

Moving on to the case where $M_H = 350\,\text{GeV}$ and $m_t = 175\,\text{GeV}$, in Fig. 4 we show the width $\Gamma(H \to t^{(*)}t^{(*)})$. $M_H = 350\,\text{GeV}$ is exactly the threshold for the Higgs to decay to on-mass-shell t quarks; and so for larger Higgs masses $\Gamma(H \to t^{(*)}t^{(*)})$ is independent of the t width, Γ_t, whereas for lower Higgs masses we expect, $\Gamma(H \to t^{(*)}t^{(*)}) \sim \Gamma_t$. For this Higgs mass between the two extremes we see a slightly reduced sensitivity to the t quark width.

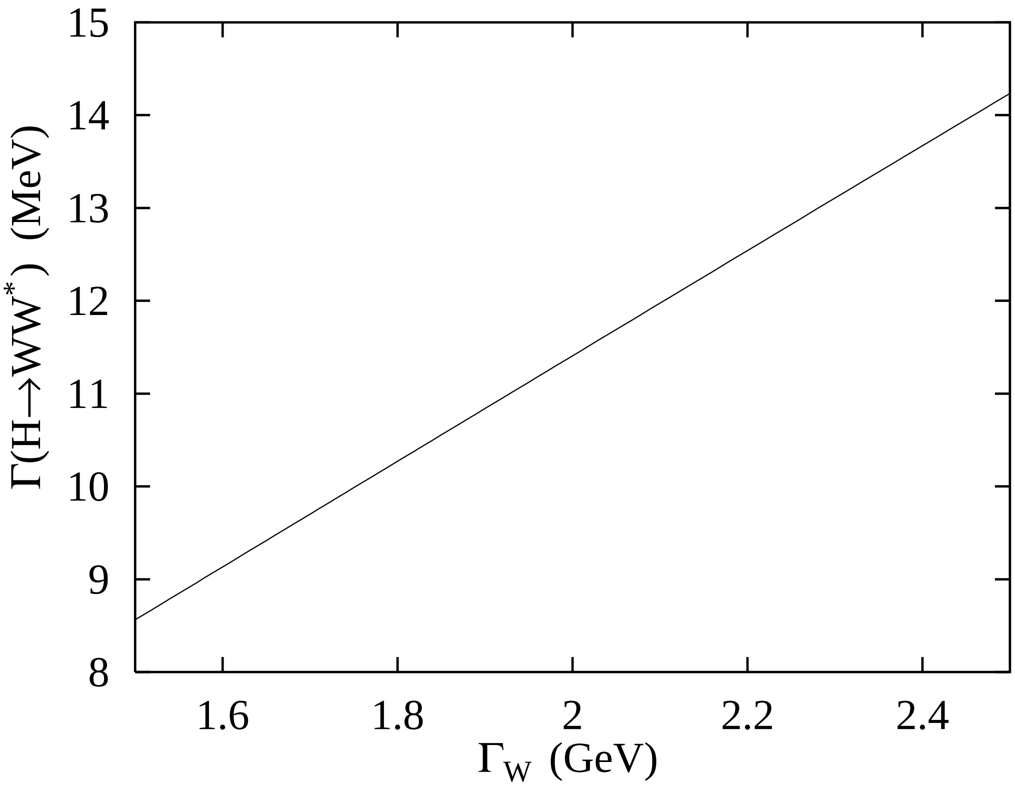

Fig. 3 The partial width $\Gamma(H \to WW^*)$ as a function of Γ_W for the case $M_H = 150\,\mathrm{GeV}$.

5 Experimental concerns

At first sight it appears that we are trading a direct measurement of Γ_W for a measurement of $\Gamma(H \to W^{(*)}W^{(*)})$, from which we indirectly extract Γ_W. It is not immediately clear why $\Gamma(H \to W^{(*)}W^{(*)})$ should be any easier to measure than Γ_W. However $\Gamma(H \to W^{(*)}W^{(*)})$ is a partial width whereas Γ_W is a total width, and ratios of Higgs decay partial widths can be measured directly from Higgs branching ratios. For example we can measure the ratio of say $\Gamma(H \to W^*W)$ to $\Gamma(H \to Z^*Z)$ by measuring the ratio of Higgs decays to WW^* to Higgs decays to ZZ^*. $\Gamma(H \to Z^*Z)$ can be accurately predicted and so we get a measurement of $\Gamma(H \to W^*W)$. Hence this method of measuring the W width is, like the CDF method, an indirect measurement, in which we need an accurate theoretical prediction of a quantity in order to be able to extract the W width. We expect that comparing $H \to WW$ to $H \to ZZ$ to be more accurate than comparing say $H \to WW$ to $H \to$ jets, or $\mathrm{Br}(H \to WW)$, as the dominant partial width of the Higgs

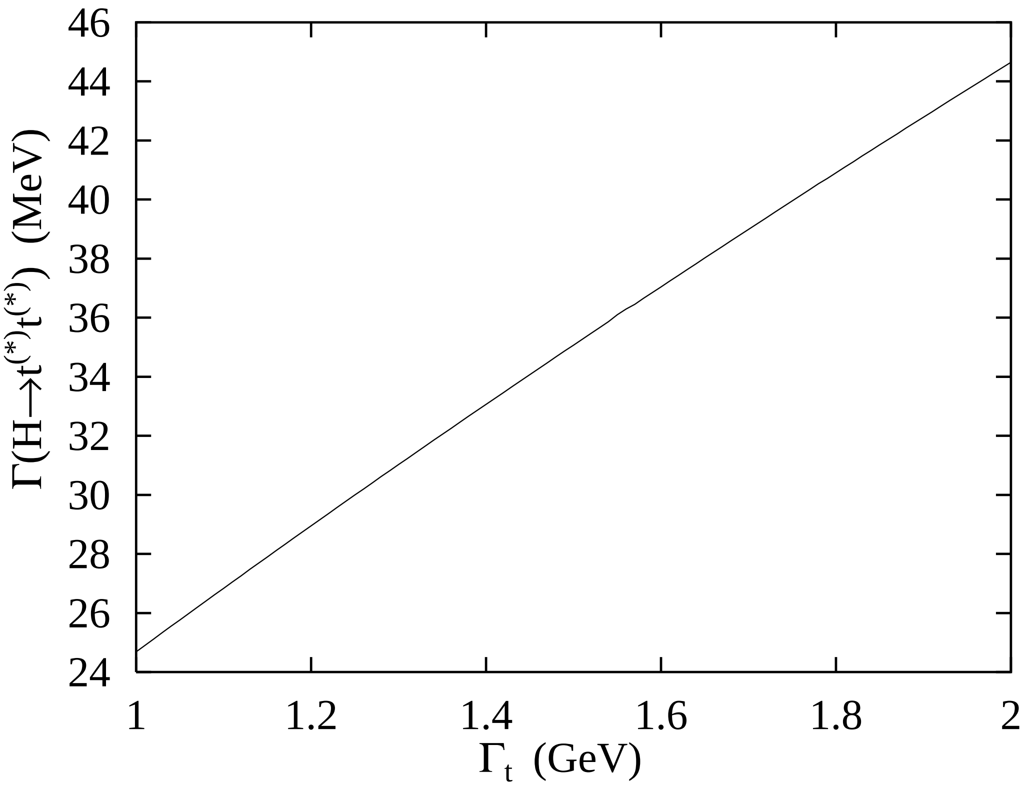

Fig. 4 The partial width $\Gamma(H \to tt^*)$ as a function of Γ_t for the case $M_H = 350\,\mathrm{GeV}$.

is for $H \to b\bar{b}$. The width $\Gamma(H \to b\bar{b})$ has a very different structure to $H \to VV$, and so for example the radiative corrections will be somewhat different. Recall that $\Gamma(H \to b\bar{b})$ is decreased by a factor of 2 from leading order to next–to–leading order, largely due to the running of the b quark mass which affects the Hbb coupling. Of course it may not be practical to accurately measure the decay $H \to ZZ^*$ as the branching ratio for $H \to ZZ^*$ is typically an order of magnitude smaller than $H \to WW^*$ for $110\,\mathrm{GeV} \lesssim H_H < 2M_W$. While we may have enough events to experimentally measure $H \to WW^*$ we will have far fewer $H \to ZZ^*$ events and so may not be able to measure the rate of these events accurately.

We now turn to ask how we will observe these Higgs events experimentally. We expect that if the Higgs boson has mass of interest for measuring massive particle widths, *i.e.*, $110\,\mathrm{GeV} \lesssim M_H < 2M_W$ and $M_H \sim 350\,\mathrm{GeV}$, will first be observed at hadron colliders. However when a Higgs is produced at a hadron collider it is usually produced in a messy environment, and this makes it hard to measure the specific properties of the Higgs. Typically one hopes just to be able to detect the Higgs in a particular decay channel.

As such there seems few prospects to measure Higgs branching ratios at future hadron colliders.

On the other hand future high energy e^+e^- and $\gamma\gamma$ colliders offer the opportunity of observing the Higgs in a far cleaner environment; where the Higgs is produced either with no other observable particles, or in a relatively simple event. For example in the processes

$$e^+e^- \to ZH \tag{5.1}$$

$$e^+e^- \to e^+e^- ZZ \to e^+e^- H \qquad , \tag{5.2}$$

the Higgs can be fairly cleanly identified by looking at the mass that recoils against either the Z or the e^+e^- pair[†]. This would peak very strongly on the Higgs mass if it were not for initial state radiation off the e^+e^- pair; in practice this radiation smears the recoil mass somewhat, however there is still a sharp peak at the Higgs mass, which can be used as a tag for Higgs events [8]. For example at a NLC with $\sqrt{s} = 300\,\mathrm{GeV}$ and $\int \mathcal{L} = 10\,\mathrm{fb}^{-1}\mathrm{year}^{-1}$ then for $M_H = 150\,\mathrm{GeV}$ we expect $\mathcal{O}(1000)$ ZH Bjorken events per year [8].

Having identified these Higgs events one can easily look at the decay products of the Higgs. If we are interested in measuring the W width then we want to be able to distinguish decays of Higgs to WW from other decays of Higgs, in particular Higgs decays to ZZ. If both W's decay hadronically then we are unlikely to be able to distinguish the W decays from Z decays; however if at least one W decays leptonically then we can distinguish the W which decays into a single observed lepton, and the Z which decays into a pair of leptons. So if $M_H = 150\,\mathrm{GeV}$ and we produce 1000 tagged Higgs we would expect to have $\mathcal{O}(250)$ WW^* events tagged by a single W decaying to an isolated lepton + missing energy; on the other hand we would only have $\mathcal{O}(20)$ ZZ^* events tagged for a single Z decaying leptonically. Thus with only $\mathcal{O}(1000)$ Higgs events we will have enough tagged WW^* events that the statistical error on the event rate is small, whereas the statistical error on the ZZ^* event rate is still relatively large. This means that with only $\mathcal{O}(1000)$ Higgs events we do better by measuring the $H \to WW^*$ branching ratio and extracting $\Gamma(H \to WW^*)$ from that.

However if we measure specific decay products of the W and Z bosons then it is only that specific width that enters in the numerator of the Higgs partial width. For example

[†] In the case where we have $e^+e^- \to ZH \to ZZZ$ notice that we can typically identify which Z does not come from the Higgs decay, as the mass that recoils against this Z is equal to M_H which is typically not true for the Z bosons from the Higgs decay.

if we observe the decay

$$\begin{array}{l} H \rightarrow W W^* \rightarrow l\nu \\ \quad\;\; \hookrightarrow \text{jets} \end{array} \qquad , \tag{5.3}$$

then the Higgs width for this process is proportional to the width $\Gamma(W \to l\nu)$ rather than the total W width; of course if we observe

$$\begin{array}{l} H \rightarrow W^* W \rightarrow l\nu \\ \quad\;\; \hookrightarrow \text{jets} \end{array} \qquad , \tag{5.4}$$

then this is proportional to $\Gamma(W \to \text{jets})$. So by measuring different off-shell decays of the Higgs we get a measurement of different partial width of that massive particle. Of course the ratio of these two rates is just the ratio of off-mass-shell W branching ratios, and we expect these to be very similar to the on-mass-shell W branching ratios which we expect will be accurately measured at LEPII.

Notice that typically for a Higgs mass in the range $110\,\text{GeV} \lesssim M_H < 2M_W$ only a single W is forced off mass shell, the other W is typically produced on mass shell and so shows no dependence on the W width.

6 Theoretical and experimental accuracy

We should ask how accurate are our theoretical predictions of these partial widths of the Higgs.

Rewriting the numerator of $\Gamma(H \to Z^{(*)}\gamma)$ and $\Gamma(H \to V^{(*)}V^{(*)})$ as Γ_V includes all the radiative corrections associated with the decay of V. This means that we are only vulnerable to radiative corrections to the production of vector bosons, and also to interference between the the decay products of the 2 vector bosons. The former corrections are only electroweak in nature, and so we expect them to be $\mathcal{O}(\alpha_{\text{em}})$ and so small; the latter we expect to be suppressed as the two vector bosons will decay at different time scales[9]. As a result we expect our results to be accurate to a few percent.

For the case $\Gamma(H \to t^{(*)}\bar{t}^{(*)})$, again rewriting the t width in the numerator effectively includes all the radiative corrections associated with the decay of the t quark, however in this case the radiative corrections to the production of a $t\ \bar{t}$ pair are QCD in nature; so we expect the radiative corrections to be $\mathcal{O}(\alpha_S)$ or 10% or so. This means that the current calculation could not be used to make accurate measurements of the t quark width. However the dependence on the t quark width that our calculation shows we would expect to be reproduced in a more accurate calculation. As such our results should be taken as

representative of the kind of accuracy that one can expect in measuring the t quark width if one had a more accurate calculation including the QCD corrections.

If there is physics beyond the Standard Model then we should worry that the decay of a Higgs to a virtual massive particle is not that predicted by the Standard Model. Indeed such physics beyond the Standard Model is likely to show up in the production of the massive particle, rather than its propagation and decay which are already measured at current colliders. Now if the production of the massive particle differs from the Standard Model prediction then we lose all ability to measure the massive particle width until that physics is understood to the level to which we wish to measure the massive particle width. However if we measure the W width by comparing the rate of $H \to W^*W$ decays to $H \to Z^*Z$ decays then as we expect the W and Z masses to be generated by the same symmetry breaking mechanism we expect that the ratio of their couplings to be independent of the physics beyond the Standard Model, that is $g_{HWW}/g_{HZZ} = M_W^2/M_Z^2$. Thus if the production rate of $H \to W^*W$ is changed we would expect to change the production rate of $H \to Z^*Z$ by a similar amount. This means that the ratio of WW^* decays to ZZ^* decays is fairly insensitive to physics beyond the Standard Model. For other ratios of Higgs decays we are not so lucky and beyond-the-Standard-Model physics will probably have drastic effects, making a measurement of massive particle widths impossible, at least until the additional physics is accurately understood.

An experimental difficulty arises if we do not know the exact Higgs mass, as the theoretical partial widths can often vary rapidly with the Higgs mass. So for example if the Higgs mass is 1% higher than 150 GeV the $\Gamma(H \to WW^*)$ partial width is 16% higher! So it seems that to measure the W width to 10% we need to know the Higgs mass to about 1 GeV. However we can decrease this sensitivity on the Higgs mass by comparing different decays of the Higgs, as we have to do if we are to measure the $H \to W^{(*)}W^{(*)}$ width. For example the $H \to Z^{(*)}Z^{(*)}$ partial width drops rapidly for lighter Higgs just like the width for $H \to W^{(*)}W^{(*)}$, so the ratio of Higgs decays to $W^{(*)}W^{(*)}$ and $Z^{(*)}Z^{(*)}$ is far less sensitive to M_H, while still retaining sensitivity to the W width. We show the ratio $\Gamma(H \to W^{(*)}W^{(*)})/\Gamma(H \to Z^{(*)}Z^{(*)})$ as a function of the Higgs mass in Fig. 5.

Clearly this ratio has far less dependence on the Higgs mass for Higgs masses less than 155 GeV, and so an accurate knowledge of M_H is not so crucial. However for Higgs masses over 155 GeV the width $\Gamma(H \to W^{(*)}W^{(*)})$ grows very rapidly as the W's start to come on mass shell, this means that even the ratio $\Gamma(H \to W^{(*)}W^{(*)})/\Gamma(H \to Z^{(*)}Z^{(*)})$ is very sensitive to the Higgs mass and so is not a good means to measure the W width. As $\Gamma(H \to Z^{(*)}Z^{(*)})$ is growing more rapidly with the Higgs mass than all other partial

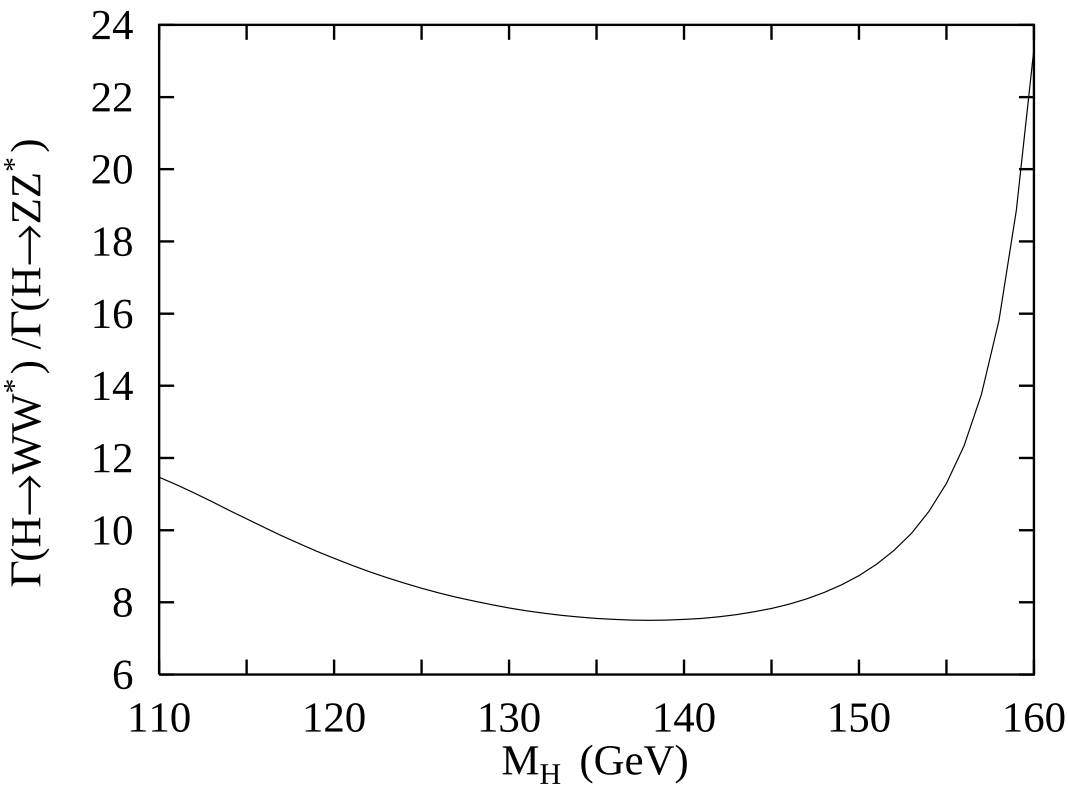

Fig. 5 The ratio of the partial width of
$\Gamma(H \to WW^*)$ to $\Gamma(H \to ZZ^*)$ as a function of M_H.

widths, with the exception of $\Gamma(H \to W^{(*)}W^{(*)})$, all other ratios of the Higgs partial widths will show far greater dependence on the Higgs mass. So we must conclude that for Higgs masses $155\,\mathrm{GeV} \lesssim M_H < 2M_W$, although the $\Gamma(H \to W^{(*)}W^{(*)})$ width is still proportional to the the W width, that measuring it is not a practical method for measuring the W width, unless the Higgs mass is known exceptionally well.

In the case where we look at measuring the t quark width the situation is clearly worse. For $M_H = 350\,\mathrm{GeV}$ the $H \to t^{(*)}t^{(*)}$ width is clearly growing very rapidly due to the narrow t quark width, and no other partial width of the Higgs shows anywhere near as rapid growth. This means that for $M_H = 350\,\mathrm{GeV}$ we must know the Higgs mass to about $0.3\,\mathrm{GeV}$ just to measure the t quark width to 10% accuracy. On the other hand for lower Higgs masses where the $H \to t^{(*)}t^{(*)}$ width grows less rapidly (but still quickly) with the Higgs mass the partial width $\Gamma(H \to t^{(*)}t^{(*)})$ is exceptionally small, also due to the narrow t quark width, and so will be exceptionally hard to detect experimentally.

7 Conclusions

In this paper we calculate the decays of the Standard Model Higgs that proceed via a massive intermediate particle. These results differ from those previously published due to a more careful treatment of the form of the Breit Wigner propagator for the massive particle.

We calculate the branching ratios of a Standard Model Higgs with the currently available values for the physical parameters.

We show that the rate for the Higgs to decay via an intermediate virtual massive particle is regulated by that massive particle's width; as such this gives a possible method for measuring massive particle widths if the Higgs happens to have a convenient mass.

We briefly discuss how one would hope to experimentally measure the partial widths of the Higgs boson at future colliders. We also discuss how the partial widths are affected by higher order corrections, and experimentally measured parameters. In particular we discuss how these partial widths to a virtual particle depend sensitively on the Higgs mass. Often this dependence is so sensitive that we can imagine that measurement of these widths will give a potentially accurate measure of the Higgs mass. However in the current case where we are interested in measuring these partial widths as a means of measuring massive particle widths, this great sensitivity on the Higgs mass reduces our ability to measure the massive particle width. This means that we either need to know the Higgs mass exceptionally accurately, or form the ratio of the massive particles partial width with another Higgs decay which shows a similar dependence on the Higgs mass. Currently there are two massive particles whose widths are not known particularly accurately, the W boson and the t quark. For the W boson we can form the ratio $\Gamma(H \to W^{(*)}W^{(*)})/\Gamma(H \to Z^{(*)}Z^{(*)})$ which is relatively insensitive to the Higgs mass if $H_H \lesssim 155\,\mathrm{GeV}$ yet still retains its sensitivity to the W width. For the t quark there is no ratio that removes the strong dependence on the Higgs mass; this, and that the $H \to t^{(*)}t^{(*)}$ drops very rapidly below the tt threshold, means that it is not practical to measure the t quark width in Higgs decays.

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